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Investigation of Plant Protection Problem through Mathematical Model in the Stationary Case

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ABSTRACT

In this paper, we investigate the problem of protecting plants of tropical agriculture at three levels of "pests of insect plants, beneficial insects". Agricultural model is formed and reasonably necessary and sufficient conditions for the existence of plant protection solutions.

Keywords: *Model, Beneficial Insects, Protection Problem, Number of Population, Stationary Case*.

INTRODUCTION

The protection of the planned agricultural crop throughout the world is one of the most important state tasks. The development of methods for protecting crops from agricultural pests naturally requires the prediction of the dynamics of biological populations, communities and ecosystems under certain anthropogenic influences. At the same time, experiments on real systems are very expensive, long and often unacceptable, so there is a need to develop various kinds of mathematical models. With the help of mathematical models it became possible to experimentally study the consequences of some planned activities affecting the functioning of natural systems, direct experiments with which are unacceptable. A number of works of domestic and foreign scientists are devoted to the problems of mathematical modeling of the dynamics of the number of biological populations. Among them, it is necessary to highlight the remarkable works of Vito Volterra, Lotka, R. May, Y. Svirezhev, Yunusi (s) M.K. and a number of other scientists. It is known that one of the main problems of agriculture is effective control of pests of agricultural crops, which includes two tasks. First, on the basis of available information on agrocenosis, pest damage thresholds are determined and the levels of effectiveness of entomo-phages are useful insects. Moreover, these parameters are determined locally by entomologists according to the calculation of a certain field (usually on plants). The results obtained are then extended to the remaining areas. The second task is the use of pesticides for pest control. It is clear that such a method of determining the parameters of an integrated control method (a combination of agro-technical, biological and chemical methods of struggle), due to lack of information and inaccuracy, will not reflect the real picture in the studied agriculture. Therefore, the task arises to formalize the process of determining the pest damage severity thresholds and the levels of effectiveness of entomology (in practice this task is usually called the preparatory task).

Consider a model agriculturist that is in equilibrium. Then the total biomasses (or numbers) of species belonging to the corresponding trophic levels satisfy the system of algebraic and differential equations [1]:

$$\begin{cases} Q + F_0(N_0, N_1) = 0 \\ N_1 F_1(N_0, N_1, \tilde{N}_1) = 0, \quad \tilde{N}_i = \int_{\alpha_i}^{\beta_i} N_i(a)da \\ \dfrac{dN_2}{da} = N_2 F_2(N_1, N_2, \tilde{N}_3), \; N_2(0) = \int_0^\infty B_2(a) N_2(a)da \\ \dfrac{dN_3}{da} = N_3 F_3(N_2, N_3), \; N_3(0) = \int_0^\infty B_3(a) N_3(a)da \end{cases} \qquad (1)$$

In general, the total biomasses (or numbers of species) belonging to the corresponding trophic levels will be denoted by N_i, $N_i = N_i(t)$, $i = \overline{0,3}$, (i = 0- resource, i = 1-plant, i = 2-harmful insects, I = 3-useful insects), Q-

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rate of external resource input, $B_i = B_i(\cdot) \geq 0 -$ function of insect birth rate.

Where the functions $F_i = F_i(\cdot) -$ Are the corresponding specific rates of growth of biological species of agrocenosis, with

$$\frac{\partial F_i}{\partial N_i} \leq 0, \quad \frac{\partial F_i}{\partial N_j} = \begin{cases} \leq 0, & i < j, \; i = \overline{0,3} \\ \geq 0, & i > j, \; j = \overline{0,3} \end{cases}$$

$\tilde{N}_i = \tilde{N}_i(t), \; i = 2,3$, the sum total of the harmful and useful insects, respectively, the amount being taken at those ages that damage the crop and destroy pests. We note that for point models $\tilde{N}_i = \tilde{N}_i(t), \; i = 2,3$.

We formulate the problem of plant protection of the Yunusi problem type in terms of stationary model aggregation (1). Let N be the planned level of the biomass of the crop, no less than which we want to preserve the crop, those

$$N_1 \geq N_1^P, \; N_1^P \in \left[N_1^{\min}, \; N_1^{\max} \right] \qquad (2)$$

where $\left[N_1^{\min}, \; N_1^{\max}\right] - const > 0.$

Consider the inequalities

$$\tilde{N}_2 \leq N_2^P, \; \tilde{N}_3 \geq N_3^P, \qquad (3)$$

where $N_2^P \geq 0, \; N_3^P \geq 0$ unknown numbers.

Definition. The value N_2^p will be called the threshold of harmfulness of harmful insects, and N_3^p the level of effectiveness of useful insects (entomophages).

Consider the case when the function $F_i(\cdot)$ is defined by the following formulas

$$\begin{cases} F_0(\cdot) = -\alpha_0 N_0 N_1 \\ F_1(\cdot) = k_0 \alpha_0 N_0 - \alpha_1 \tilde{N}_2 - m_1 \\ F_2(\cdot) = k_1 \alpha_1 N_1 - \alpha_2 \tilde{N}_3 - m_2 \\ F_3(\cdot) = k_2 \alpha_2 N_2 - \varepsilon N_3 - m_3 \end{cases} \qquad (4)$$

those. The interaction of species is described according to Voltaire's law, which can be considered fair in the case of "impending" tropic connections. This means that food for pests is abundant in useful insects that feed only on pests.

The increase in the number of pests in a short period of time is proportional to the product of the biomass of agricultural crops by the number of pests, the growth of useful insects is also proportional to the product of the number of beneficial insects by the number of pests, and the natural mortality of insects is proportional to their number. The specific growth rate denotes: $m_i -$ averaged coefficients of natural mortality, $i = 1,2,3; \; k_i -$ share of consumed biomass, going for reproductive exchange and growth; $i = 0,1,2; \; \alpha_i -$ coefficients of trophic functions, $i = 0,1,2; \; \varepsilon -$ coefficient of self-limited population of useful insects.

The following theorem on the existence of a solution of the stationary plant protection problem is valid.

THEOREM. Let the interactions between the types of agrocenosis proceed according to Voltaire's law (that is, according to the formulas (4)), then in order for

$$N_1 \geq N_1^P, \; N_1^P \in \left[N_1^{\min}, \; N_1^{\max}\right],$$
$$N_1^{\min} = \frac{m_2}{k_1 \alpha_1}, \quad N_1^{\max} = \frac{k_0 Q}{m_1}$$

It is necessary and sufficient that the following inequalities are satisfied:

$$\begin{cases} N_0 \leq \dfrac{Q}{\alpha_1 N_1^P}, \\ \tilde{N}_2 \leq N_2^P, \quad N_2^P = \dfrac{k_0 Q}{\alpha_1 N_1^P} - \dfrac{m_1}{\alpha_1} \\ \tilde{N}_3 \geq N_3^P, \quad N_3^P = \dfrac{k_1 \alpha_1}{\alpha_2} N_1^P - \dfrac{m_2}{\alpha_2} \end{cases} \qquad (5)$$

Necessity: Let $N_1 \geq N_1^P, \; N_1^P \in \left[N_1^{\min}, \; N_1^{\max}\right].$

Let us show the validity of (5). By virtue of (1), (4) we have:

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$$\begin{cases} Q-\alpha_0 N_0, \;\; N_1 = 0, \\ k_0\alpha_0 N_0 - \alpha_1 \tilde{N}_2 - m_1 = 0, \\ \dfrac{dN_2}{da} = N_2\left(k_1\alpha_1 N_1 - \alpha_2 \tilde{N}_3 - m_2\right), \\ \dfrac{dN_3}{da} = N_3(k_2\alpha_2 N_2 - \varepsilon N_3 - m_3) \end{cases} \qquad (6)$$

From the first equation (6) with allowance for $N_1 \geq N_1^P$, we obtain $Q - \alpha_0 N_0 N_1 = 0$ i.e.

$$N_0 \leq \frac{Q}{\alpha_1 N_1^P} = \tilde{N}_0.$$

From the second equation (6) we have:

$$k_0\alpha_0 \frac{Q}{\alpha_1 N_1^P} - \alpha_1 \tilde{N}_2 - m_1 \geq 0, \;\; \tilde{N}_2 \leq \frac{k_0 Q}{\alpha_1 N_1^P} - \frac{m_1}{\alpha_1} = N_2^P.$$

Similarly, from the third equation (6) we obtain:

$$\frac{dN_2}{da} = N_2(k_1\alpha_1 N_1 - \alpha_2 \tilde{N}_3 - m_2)$$

both sides of the equation, multiplying by $N_2(a)$ and integrating the result with respect to a, we get:

$$\int_0^\infty N_2(a)\frac{dN_2}{da}da = \int_0^\infty N_2^2(a)da\left[k_1\alpha_1 N_1 - \alpha_2 \tilde{N}_3 - m_2\right]$$

$$\frac{1}{2}N_2^2(\infty) - \frac{1}{2}N_2^2(0) = \int_0^\infty N_2^2(a)da\left[k_1\alpha_1 N_1 - \alpha_2 \tilde{N}_3 - m_2\right]$$

Hence

$$k_1\alpha_1 N_1 - \alpha_2 \tilde{N}_3 - m \leq 0, \;\; \tilde{N}_3 \geq \frac{k_1\alpha_1}{\alpha_2} N_1^P - \frac{m_2}{\alpha_2}.$$

We estimate N_3.

From the fourth equation (6) we obtain:

$$N_3(a) = \frac{N(0)e^{\int_0^\alpha A(\xi)d\xi}}{1 + \varepsilon N(0)\int_0^\alpha e^{\int_0^\alpha A(\xi)d\xi} d\xi} \leq N_{\max},$$

where $A(a) = k_1\alpha_1 N_2 - m_3.$

$$\frac{dN_3}{da} = A(a)N_3(a) - \varepsilon N_3^2(a),$$

Really,

$$\frac{dN_3}{da} = A(a)N_3(a) - \varepsilon N_3^2(a),$$ i.e.

$$-\frac{1}{N_3^2}\frac{dN_3}{da} = -\frac{A(a)}{N_3} + \varepsilon.$$

Hence, introducing the notation $\dfrac{1}{N_3(a)} = y,$ we have $y'(a) = -A(a)y + \varepsilon,$ it is easy to see that

$$y(a) = y(0)e^{-\int_0^a A(\xi)d\xi} + \varepsilon\int_0^a e^{-\int_0^a A(\xi)d\xi} d\xi,$$

and therefore,

$$\frac{1}{N_3(a)} = \frac{1}{N_3(0)}e^{-\int_0^a A(\xi)d\xi} + \varepsilon\int_0^a e^{-\int_0^a A(\xi)d\xi} d\xi,$$

as required.

Adequacy. Suppose that inequalities (5) hold. We show that

$$N_1 \geq N_1^P, \;\; N_1^P \in \left[\frac{m_i}{k_1\alpha_1}, \frac{k_0 Q}{m_1}\right].$$

From the third equation (6), taking (5) into account, we have $k_1\alpha_1 N_1 - \alpha_2 \tilde{N}_3 - m_2 \geq 0,$

$k_1\alpha_1\left(N_1 - N_1^P\right) \geq 0$ from here $N_1 \geq N_1^P.$

An analogous result is obtained on the basis of the use of the first and second equations (6). Since in (5) the quantities $N_2^P, \; N_3^P$ are nonnegative, then

$$\frac{m_2}{k_1\alpha_1} \leq N_1^P \leq \frac{k_0 Q}{m_1}, \;\; Q \geq \frac{m_1 m_2}{k_0 k_1 \alpha_1}.$$

EXAMPLE. Let at some field we want to keep the harvest of agricultural crops no less than conventional units and $N_1^P = 45$, Q = 5500 conventional units,

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$$k_0 = 0.9,\ k = 0.8,\ m_1 = m_2 = 0.02,\ \alpha_1 = \alpha_2 = 1,$$

then we define the quantity $N_2^P, u \ \ N_3^P$ by the following formulas

$$N_2^P = \frac{k_0 Q}{\alpha_1 N_1^P} - \frac{m_1}{\alpha_1},$$
$$N_3^P = \frac{k_1 \alpha_1}{\alpha_2} N_1^P - \frac{m_2}{\alpha_2}. \qquad (7)$$

It follows from (7) that $N_2^P \approx 110,\ N_3^P \approx 36$ Therefore, for these outputs, in order to obtain at least 45 conventional units of the crop, the number of pests should be no more than 110 units, and the number of useful insects is not less than 36 units. With such proportions between the numbers of harmful and useful insects in agrocenosis, a state occurs, in which there is no need to use chemical agents against pests. We note that the results obtained qualitatively coincide with the empirical scales of the staff of the Institute of Zoology and Parasitology of the Academy of Sciences of the Republic of Tajikistan.

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